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NOTES ON THE ELEMENTARY ALGEBRAIC THEORY OF A CLASS OF PHOTOGRAPHIC OBJECTIVES

BY E. T. HANSON, B.A.

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ABSTRACT. After a short discussion of the general theory of the first order aberrations in a system of thin lenses, the equations expressing the absence of Seidel's first four aberrations in an objective consisting of two thin systems of lenses, separated by an interval, are formed. When each of the two thin systems consists of only two lenses, the equations can be put in a form which admits of an elegant graphical solution, regard being paid to the necessity of obtaining a solution in which no one of the lens curvatures exceeds a certain limit.

Introduction.

THE application of the theory of the first order aberrations to the direct computation of an optical system is limited to a few simple cases. The most satisfactory application refers to photographic objectives. The design of a system which will satisfy the theory, while meeting practical requirements, is not easy. This investigation endeavours to present a purely graphical method of arriving at the design of an objective which will satisfy Seidel's first four conditions. These are the conditions for absence of spherical aberration, coma, astigmatism, and curvature of the field. The condition for achromatism is also taken into account.

It is given as an example of a graphical solution which is believed to be new, and with the hope that it may suggest further applications.

It does not seem to have been noticed that the equations can be put in a very simple form when the objective considered consists of two thin systems of lenses separated by an interval, thus making the mathematical treatment more elegant than in its usual presentation.

In the following notes is first given the proof of the general form which the equations take. When each of the two thin systems consists of two single lenses, the method is useful in that it quickly shows up cases when a solution fails. It has been of some interest to investigate a number of cases, and to follow the transformations to a case which is eminently suitable from a practical standpoint.

General Theory.

In the following application of the first order theory to photographic objectives all the lenses will be assumed to possess the same refractive index though they may differ in dispersive power. An extension of the method to cases in which the refractive index is not the same throughout is given at the end of the paper.

The conditions for absence of the four aberrations, curvature of the field, astigmatism, coma, and spherical aberration, for all positions of the stop, can be written as follows*:

$$\left.\begin{aligned} S \text{ (Spherical aberration)} &= \Sigma H_s^4 j_s^3 F_s &= 0 \\ C \text{ (Coma)} &= \Sigma H_s^4 j_s^3 E_s F_s &= 0 \\ As \text{ (Astigmatism)} &= \Sigma H_s^4 j_s^3 E_s^2 F_s &= 0 \\ \kappa \text{ (Curvature)} &= \Sigma \rho_s \kappa_s &= 0 \end{aligned}\right\} \quad \ldots\ldots(1).$$

The notation requires explanation. Consider that portion of a paraxial ray which is incident upon the sth surface. If this ray is incident at a distance h_s from the optical axis, then

$$H_s = h_s/h_1.$$

Let n_s, n_s' be the refractive indices in front of and behind the sth surface respectively. Let ρ_s be the reciprocal of the radius, and ω_s the reciprocal of the distance from the surface at which the ray cuts the axis. Then

$$j_s \text{ (the zero invariant)} = n_s(\rho_s - \omega_s).$$

Let
$$\frac{1}{n_s'} - \frac{1}{n_s} = \kappa_s \text{ and } \frac{1}{n_s'^2} - \frac{1}{n_s^2} = \lambda_s.$$

Then
$$F_s = \rho_s \kappa_s - j_s \lambda_s.$$

If t_q $(q < s)$ be the distance between the qth and the $(q+1)$th surfaces, then

$$E_s = \sum_{q=1}^{q=s-1} \frac{t_q}{n_q' H_{q+1} H_q} + \frac{1}{H_s^2 j_s}.$$

Consider an optical system comprising a number of co-axial members separated by finite intervals, each member being a thin system of lenses. Let the members be numbered in succession from the front 1 to p, p being the last member. Let the interval separating the $(p-1)$th and the pth members be t. Denote that part of the system comprising the first $(p-1)$ members by A, and the pth member by B.

* See Southall's *Principles of Geometrical Optics*, 2nd ed. p. 466; the first four of equations 356, when equated to zero, are equivalent to (1): or Whittaker's *Theory of Optical Instruments*, p. 44.

Let $$\Sigma \frac{t_q}{n_q' H_{q+1} H_q} = T_1,$$

the summation being taken over the first $(p-1)$ members. Then, if s now denote a surface in B,

$$E_s = T_1 + \frac{t}{H_{p-1} H_p} + \frac{1}{H_s^2 j_s}.$$

Let $$T = T_1 + t/H_{p-1} H_p,$$

and let S_1, C_1, As_1 be the coefficients of spherical aberration, coma, and astigmatism respectively for A. Let S_2, C_2 be the coefficients of spherical aberration and coma for B.

Now, if Petzval's condition for absence of curvature is satisfied and if the refractive index is uniform throughout,

$$\Sigma F_s \text{ for } B = -\Sigma F_r \text{ for } A,$$

if r denote a surface in A.

On the left-hand side the summation is taken over the pth member only, and on the right-hand side it is taken over the first $(p-1)$ members only.

Let, therefore,

$$As_1 - \Sigma F_r \text{ for } A = a_1,$$

so that a_1 belongs only to A.

Then, for the removal of spherical aberration, coma, and astigmatism,

$$\left.\begin{aligned} S_1 + H_p^4 S_2 &= 0 \\ C_1 + H_p^4 T S_2 + H_p^3 C_2 &= 0 \\ a_1 + H_p^4 T^2 S_2 + 2H_p^3 T C_2 &= 0 \end{aligned}\right\} \quad \ldots\ldots(2).$$

H_p is, of course, common to every surface in the pth member. From these equations can be obtained

$$S_1 T^2 - 2C_1 T + a_1 = 0,$$

so that a solution is possible only if

$$C_1^2 - a_1 S_1 > 0.$$

If A consists of a single thin system of lenses,

$$a_1 = 0 \text{ and } T = t/H,$$

where H refers to, and is common to, every surface of member B. Introduce a variable quantity θ, then equations (2) reduce to

$$\left.\begin{aligned} S_1 t - 2HC_1 &= 0 \\ S_2 t + \frac{2}{H} C_2 &= 0 \\ -C_1 + H^2\theta &= 0 \\ -C_2 + \theta &= 0 \end{aligned}\right\} \quad \ldots\ldots(3).$$

These equations are equivalent to the general equations for the removal of Seidel's first three image defects in an objective consisting of two thin systems of lenses separated by an interval, the refractive index being taken as uniform and the simple condition being satisfied for removal of the Petzval curvature.

If the focal length of the back member be taken as unit of length, then that of the front member is equal to -1, distances being positive when measured in the direction of the incident light. In this case

$$H = 1 + t.$$

The Solution of the Equations.

If the surfaces of a thin lens be designated by the numbers 1 and 2, and if ϕ_2 be the reciprocal of the focal length, then

$$F_1 + F_2 = \phi_2 \text{ and } j_1 - j_2 = \phi_2/\kappa_2.$$

Let ω_2' be the reciprocal of the second conjugate focal length and let S and C be respectively the coefficients of spherical aberration and coma.

Then the following abbreviations are useful:

$$l = -\frac{n}{3n+2}, \qquad m = -\frac{n}{n+1},$$

$$p = \frac{n}{2(n-1)(3n+2)}, \quad q = \frac{n}{n-1},$$

$$N = \frac{ql}{m^2},$$

where n is the refractive index.

Let a_2, c_2 and b_2 be defined as follows:

$$\left.\begin{aligned} a_2 &= p\phi_2 - l\omega_2' \\ c_2 &= q\phi_2 - m\omega_2' \\ b_2 &= \phi_2 N c_2 - a_2^2 \end{aligned}\right\} \qquad \text{.....(4).}$$

The equations for S and C then become

$$\left.\begin{aligned} lS/\phi_2 &= (j_2 + a_2)^2 + b_2 \\ \text{and} \qquad mC/\phi_2 &= j_2 + c_2 \end{aligned}\right\} \qquad \text{.....(5).}$$

An interesting alternative expression for the spherical aberration of a thin lens is

$$S = C(j_1 + j_2) - j_1 j_2 \phi_2,$$

so that if a thin lens is corrected for coma,

$$S = -\phi_2 j_1 j_2.$$

It will be seen from (4) that a simple set of charts will greatly facilitate the calculation of a_2, c_2, and therefore b_2 in any particular case. For if ϕ_2 and ω_2' be measured along rectangular co-ordinates, the curves $a_2 =$ const. and $c_2 =$ const. are simply two families of parallel straight lines. For the purpose of the immediate

problem, the value of n taken was 1·6. Against each line of one family the values of a_2 and a_2^2 were marked. Against each line of the other family the values of c_2 and Nc_2 were marked. Two such charts were constructed and proved very useful.

They are convenient in giving quickly certain quantities which are independent of the coflexure of the lens, when a number of cases have to be considered numerically. Returning now to the objective consisting of two thin systems of lenses separated by an interval, let the numerals, 2, 4, 6, 8, etc. represent each lens in succession from the front throughout the whole objective. The following notation is useful:

It is to be understood that, for all the component lenses of member A, g and ψ are defined by the equations

$$\left.\begin{aligned} g &= -\frac{H}{t}\cdot\frac{l}{m} \\ \text{and} \quad \psi &= H^2 m\theta \end{aligned}\right\} \qquad \text{.....(6),}$$

and that, for all the component lenses of member B,

$$\left.\begin{aligned} g &= \frac{1}{Ht}\cdot\frac{l}{m} \\ \text{and} \quad \psi &= m\theta \end{aligned}\right\} \qquad \text{.....(7).}$$

Omitting suffixes for the present, define J, β, and α as follows:

$$\begin{aligned} J &= j + a + g, \\ \beta &= c - a - g, \\ \alpha &= b + 2g\beta + g^2. \end{aligned}$$

Equations (3) may now be written

$$\left.\begin{aligned} \Sigma\phi\,(J^2 + \alpha) &= 0 \\ \Sigma\phi\,(J + \beta) &= \psi \end{aligned}\right\} \qquad \text{.....(8).}$$

These represent four equations since the summations are taken only over one member at a time, keeping (6) and (7) in mind. Write

$$\Sigma\phi\alpha = M,$$

e.g. if there are three lenses in A,

$$\Sigma\phi\alpha = \phi_2\alpha_2 + \phi_4\alpha_4 + \phi_6\alpha_6.$$

Write $\Sigma\phi\beta = N.$

Then, if each member now consist of two thin lenses, the first equation of (3) represents an ellipse or hyperbola referred to rectangular axes J_2 and J_4, the squares of whose semi-axes are equal to $-M/\phi_2$ and $-M/\phi_4$. If ϕ_2 and ϕ_4 are of the same sign the curve is an ellipse; if of opposite sign a hyperbola.

The third equation of (3) represents a straight line referred to the same axes, whose intercepts on the axes are

$$(\psi - N)/\phi_2 \text{ and } (\psi - N)/\phi_4.$$

The results for the second and fourth of equations (3) referred to axes J_6 and J_8 are precisely similar.

If a system be given in which the ϕ's and t are known, the whole problem is solved by choosing the value of θ, which is a parameter of the straight lines only, in such a way that the intersections of the straight lines with their corresponding ellipses or hyperbolas fall within rectangles bounded by the limits which are permissible in the values of the J's, which are linear functions of the coflexures. All the parameters are quickly calculated, for, the ratios ϕ_4/ϕ_2 and ϕ_8/ϕ_6 being known, the values of M and N are readily determined by the use of the charts.

There is no advantage in endeavouring to treat the problem generally any further, but the restrictions will be made as few as possible.

It is convenient to make one postulate, namely that there is a numerical relationship between the back focal length and the diameter D of the aperture. This ratio u_8'/D will be taken as 6.

From practical considerations, if reasonable field of view is required, t/D should not be greater than unity.

Let f be the focal length of member B, which is positive. Then $-f$ is the focal length of member A. The following equations are satisfied (remembering that $\omega = 1/u$):

$$\omega_8' - \omega_5 = 1/f,$$
$$\omega_4' = -1/f,$$
$$u_4' = u_5 + t,$$

whence
$$\left(\frac{f}{D}\right)^2 + \frac{f}{D}\cdot\frac{t}{D} - \frac{u_8'}{D}\cdot\frac{t}{D} = 0.$$

It is easily seen from this equation that, if t/D is restricted to values less than 1, f/D (the positive value only being admissible) is restricted to values less than 2. But f/D cannot be made much less than 2, otherwise the assumptions, upon which the whole theory rests, will be contravened.

The value chosen for f/D is 1·71, whence $t/D = \cdot 68$. This makes $t/f = \cdot 4$. If f is unity, as previously, then $t = \cdot 4$.

The relative focal lengths of the lenses composing the two members are so far arbitrary, being subject to the Petzval condition that the sum of the powers of those composing member A is equal to -1, and of those composing $B + 1$. Now these focal lengths require to be known before the graphical method can be applied. There is not much to guide the choosing of them except that imposed by the condition for achromatism. A number of cases have been graphically investigated.

For this purpose the unit has been altered, the separation t being now unit of length, which makes $\phi = 1/f = \cdot 4$.

Case (*a*). In this case

$$\phi_2 = \phi_4 = -\cdot 2 \quad \text{and} \quad \phi_6 = \phi_8 = +\cdot 2.$$

The constants being readily evaluated with the use of the charts, the equations to be satisfied are

$$\left.\begin{aligned} J_2^2 + J_4^2 &= (1\cdot 02)^2 \\ J_2 + J_4 &= -1\cdot 58 - 10\theta \\ J_6^2 + J_8^2 &= (\cdot 64)^2 \\ J_6 + J_8 &= 1\cdot 33 + 5\theta \end{aligned}\right\} \qquad \ldots\ldots(9).$$

The limits to the coflexures of the lenses are set by the condition that no radius must be less than 1·25. On this understanding

J_2 may lie between $-\cdot29$ and $+\cdot91$;
J_4 „ „ $-\cdot14$ „ $+1\cdot06$;
J_6 „ „ $-\cdot40$ „ $+\cdot80$;
J_8 „ „ $-\cdot55$ „ $+\cdot65$.

When referred to axes J_2 and J_4, the first of equations (9) represents a circle, and the second a family of parallel straight lines. The third and fourth have the same representation referred to axes J_6 and J_8. In Fig. 1 one of the limits between

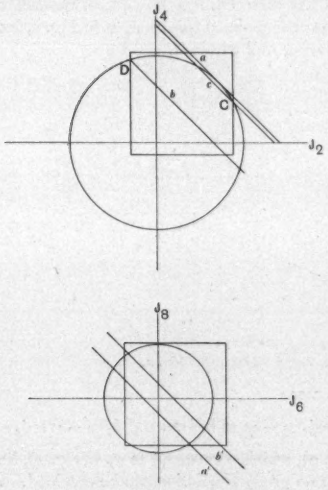

Fig. 1.

which θ may lie is found by drawing one of the family of straight lines (a) tangent to that portion of the circle which lies within the coflexure rectangle. It is obvious from the figure that the other limit of θ is found by drawing one of the family (b) through the point D, where the circle intersects the line representing the lower limit for J_2 given above. Between the tangent and the parallel line through C, where J_2 has its upper limiting value, there are two possible solutions.

Corresponding to a and b for member A are the lines a' and b' for member B. In this case it is evident that one or more of the lenses must be near its limit of coflexure. Furthermore, the requirement of achromatism necessitates the use of

glasses of widely different dispersive powers. Taking this case as a starting point, the result of increasing both ϕ_3 and ϕ_4 in absolute magnitude will be investigated. The result of decreasing them can be carried out in an exactly similar way, so that all possible cases will be included.

Considering ϕ_3 first, the circle becomes an ellipse which changes to a hyperbola as ϕ_3 passes through the value ·4. It is interesting to note that the vanishing of the discriminant is given by the equation

$$1{\cdot}170\phi_3^2 + {\cdot}027\phi_3^2 - {\cdot}200\phi_3 - {\cdot}053 = 0.$$

This has a root for $\phi_3 = {\cdot}5$, which is the only root of any value. For exactly this value of ϕ_3 the hyperbola reduces to two straight lines, so that all solutions for member B are real. As these changes take place, the coflexure rectangle moves down towards the lower right-hand quadrant, as in Fig. 2, one of the straight lines in the final position lying well within it.

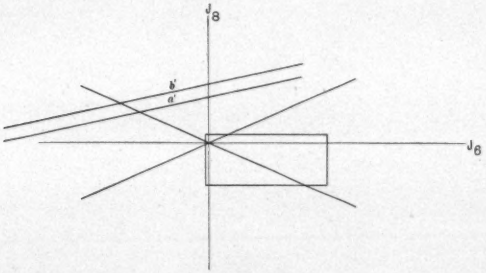

Fig. 2.

Case (*b*). In this case

$$\phi_2 = \phi_4 = -{\cdot}2 \text{ as before, and } \phi_5 = -{\cdot}1,\ \phi_3 = {\cdot}5.$$

The lines a', b' (Fig. 2), which correspond to a, b (Fig. 1), lie a long way beyond the rectangle, so that no practical solution can be obtained.

Considering now the result of increasing ϕ_4, the vanishing of the discriminant leads to nothing useful. The coflexure rectangle moves up into the top left-hand quadrant. As ϕ_4 approaches the value $-{\cdot}6$, a branch of the hyperbola passes well within the rectangle. It is now only necessary that a suitable value of θ can be found.

Case (*c*). This is shown in Fig. 3. The ordinates in this figure have been increased in the ratios $\sqrt{|\phi_4/\phi_2|}$ and $\sqrt{|\phi_3/\phi_5|}$ respectively. In this case

$$\phi_2 = +{\cdot}2,\ \phi_4 = -{\cdot}6,\ \phi_5 = -{\cdot}1, \text{ and } \phi_3 = +{\cdot}5.$$

The limits of θ are given by the lines a' and b'. To these correspond the lines a and b. If θ be now given the value $-{\cdot}69$, the lines c and c' are obtained. These

cut the two curves in P and P', which both lie well within their respective rectangles. The solution is now complete in every respect, and the coflexures of the lenses are given directly by the co-ordinates of P and P'. It may be noticed that this system is very easily achromatized with glasses of almost exactly the same refractive index. The ϕ's may now be varied slowly in any way desired, and the alterations in the J's are easily followed upon the figures.

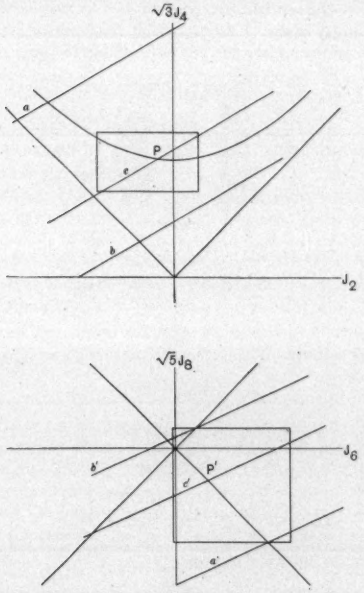

Fig. 3.

With the help of Fig. 3 it is now easy to calculate the radii of all the eight surfaces. With $\theta = -\cdot 69$, remembering that the separation of the two thin systems is the unit of the length, they are from the front

$r_1 = +\ 2{\cdot}94,$ $\quad r_5 = -\ 2{\cdot}70,$
$r_2 = -\ 16{\cdot}67,$ $\quad r_6 = -\ 5{\cdot}88,$
$r_3 = -\ 1{\cdot}79,$ $\quad r_7 = +\ 1{\cdot}89,$
$r_4 = +\ 1{\cdot}56,$ $\quad r_8 = -\ 2{\cdot}13.$

Fig. 4 gives a diagram of the objective. This diagram is obviously only approximate, since in the theory the thicknesses of the component lenses have been neglected. The aperture has been reduced in the ratio of 4 to 5 approximately, in order to make it possible to place the components of each thin system closer to one another. The field of view is accordingly somewhat more restricted.

It is interesting to note that the third lens from the front is of great focal length, and has accordingly little effect upon the aberrations. It can, therefore, be dispensed with, and all the conditions can be satisfied by making small modifications in the three remaining lenses. It is remarkable that a three-lens objective can be obtained in which all the surface curvatures lie within the prescribed limits.

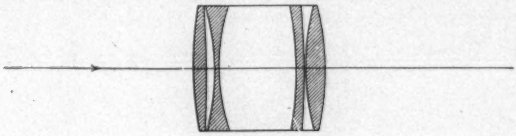

Fig. 4.

Extension of the Method to Cases in which the Refractive Index is not the same throughout.

It has been assumed in the foregoing theory that the index of refraction is uniform throughout. A difference in refractive index can, however, be allowed for in the following manner. When it is uniform throughout it has been shown that

$$-\Sigma\rho_s\kappa_s = \frac{1}{n}(\phi_2 + \phi_4 + \ldots\ldots),$$

but quite generally in a system of thin lenses

$$\Sigma F_s = (\phi_2 + \phi_4 + \ldots\ldots).$$

In the general case

$$\left.\begin{aligned} -\Sigma\rho_s\kappa_s &= \frac{1}{n_2}\phi_2 + \frac{1}{n_4}\phi_4 + \ldots\ldots \\ &= +\frac{1}{n}(\phi_2 + \phi_4 + \ldots\ldots) \\ &\quad -\frac{\Delta n}{n^2}\left[\phi_2\frac{\Delta n_2}{\Delta n} + \phi_4\frac{\Delta n_4}{\Delta n} + \ldots\ldots\right] \end{aligned}\right\} \quad \ldots\ldots(10),$$

where n is the lowest refractive index and Δn the greatest difference. In this case one of the coefficients of the ϕ's in the square brackets is unity and all the others are less than unity. Hence, if

$$\phi_2 + \phi_4 + \ldots\ldots = 0,$$

the last line of (10) is equal to

$$\pm\frac{\Delta n}{n^2}\epsilon\phi',$$

where ϵ is positive and less than unity and ϕ' is the sum of the powers of all the positive lenses. ϵ is usually considerably less than unity, for it is the difference of two positive ratios each of which is less than unity. The assumption, therefore, that $\phi_2 + \phi_4 + \ldots\ldots$ is equal to zero in the general case is very nearly equivalent to the condition for absence of the Petzval curvature, unless ϕ' is large. This does not, however, occur in practice. The foregoing graphical method may, therefore, be used in the general case when the refractive indices are different, but before it can be applied the indices must be allotted to each lens, and the charts, which have been described, must be amplified so that a_2 and c_2 can be read off for a range of refractive indices.

Summary.

The type of photographic objective particularly considered consists of two thin systems of lenses separated by an interval. Each system comprises two lenses. The objects attained in the paper are:

(1) The formulation of equations equivalent to those which satisfy Seidel's first four conditions in such a way that they may be suitable for graphical solution.

(2) The construction of simple charts which enable a number of cases to be rapidly adapted to immediate graphical treatment.

(3) The adaptation of the graphical method in such a manner that it may be readily seen when solutions of practical value can be obtained.

DISCUSSION.

Comdr. T. Y. Baker: It is interesting to note that of recent years there has been a change in the manner in which optical first order aberrations are dealt with. It is not very long ago that one might have searched in vain through optical literature for any reasoned method that could be used to treat aberrations from the designer's point of view. There were plenty of formulae for the aberrations in terms of image and stop positions and lens shapes, but no clue to the means whereby the suitable values could be found for coflexures in order to eliminate aberrations from the complete system.

Recently there have been papers by Mr Smith on this special point and there is now Mr Hanson's paper treating from the same aspect the design of a class of photographic objectives. The graphical method that Mr Hanson employs is convenient and satisfactory. Obviously, since first order aberrations alone are dealt with, a high degree of accuracy is not essential, for modifications of lens shapes must subsequently be made to correct the higher aberrations. The method of solution is especially valuable in that it is evident immediately to the eye whether the solutions obtained give lens shapes of practicable form.

It is to be hoped that these recent papers will stimulate other authors to put forward papers of a similar character.

Mr T. Smith: Mr Hanson's paper is an important contribution to the literature on the design of optical systems, and, owing to the more direct appeal inherent in a graphical method of treating mathematical problems, it will doubtless be highly and widely appreciated, especially perhaps by those who find a concise algebraic discussion somewhat difficult to follow. The limitation of the problem by the insertion of reasonable limits is a consideration of practical, almost indeed of commercial, rather than of theoretical, importance; and those engaged on lens design in industry should appreciate the importance of a method which shows without much labour whether an acceptable solution along particular lines is feasible. In deriving the construction the equations to be satisfied have been simplified in a manner which seems to depend upon the type of system considered. It would be interesting to know whether the method can be applied as readily to systems in which some of these special conditions are not satisfied, for instance, a system in which the sum of all the powers is not zero or approximately zero. It should be observed that the condition for the removal of distortion, the last of von Seidel's conditions, had not been considered, and with this particular solution it seemed necessary to leave it to take care of itself. As the image is otherwise fully corrected, it is useless to attempt to control distortion by placing a stop in a particular position. The insertion of this distortion condition in a system consisting of a positive and a negative component well separated from one another may easily prove to be undesirable practice.

Mr Hanson: I can do little more than thank Commander Baker and Mr Smith for their appreciative remarks. It is not easy to treat the design of optical systems in a general manner, and to deduce results that are of practical application. Under the circumstances I have been able to deal only with a very restricted system, having always in view the practical and perhaps, according to Mr Smith, the commercial aspect. I have not had the opportunity to consider the application of the method to systems in which the special conditions are not satisfied, but I think that it might be applied to systems in which the Petzval condition is only approximately realized. With regard to the removal of distortion it will be noted that with four lenses there is an infinite number, within limits, of cases in which the first four conditions are satisfied. A number of these cases can be readily obtained from the graphs. The distortion condition may be difficult to satisfy if the Petzval condition is exactly satisfied, so that the extension to other systems, as suggested by Mr Smith, would be useful. But, as it stands, it will not entail much more than a consideration of a few cases to find those modifications which will tend to a system approaching a realization of Seidel's fifth condition, provided that in each case some method of calculating the distortion be employed.

A GENERAL SURVEY OF THE THIN DOUBLE LENS

By T. SMITH, M.A., F.Inst.P.
(Optics Department, National Physical Laboratory.)

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ABSTRACT. The shapes of the lenses and the types of glass suitable for the construction of an objective free from all first order aberrations in a primary plane for an infinitely distant object are found on the assumption that the system is composed of two separated thin lenses, each of which consists of glasses cemented together. The analysis indicates that "old" achromats should be used for both component lenses, a conclusion not in accordance with modern practice.

In an earlier paper* general formulae have been given showing the conditions appropriate for the several lenses of a system to enable the entire instrument to satisfy assigned aberrational conditions. The simplest systems, consisting of a number of thin lenses in contact, are mathematically equivalent to a thin lens, and some of their aberrations are not under the control of the designer. The next simplest type of system is obtained by placing two sets of grouped thin lenses at an appreciable distance apart. The equations of the earlier paper are immediately applicable to such a system, which is of interest as the idealized prototype of many modern photographic objectives. The information derivable from the theory, notwithstanding all the deficiencies inherent in a treatment which considers only first order aberrations, will be generally recognized as of prime importance.

It is not necessary to introduce the powers of the lenses specifically into the equations, the important quantities being the ratios the separation between the lenses bears to their focal lengths. Denote these by $1 - \alpha_1$ and $1 - \alpha_2$ for the first and second lenses respectively. Then the Petzval coefficient ϖ for the whole system is expressed in terms of those of the components by

$$(1 - \alpha_1\alpha_2)\,\varpi = (1 - \alpha_1)\,\varpi_1 + (1 - \alpha_2)\,\varpi_2.$$

It is convenient to denote by W the value of

$$(1 - \alpha_1)\,(\tfrac{1}{2} + \tfrac{1}{3}\varpi_1) + (1 - \alpha_2)\,(\tfrac{1}{2} + \tfrac{1}{3}\varpi_2).$$

Following the notation of the previous paper as modified in the footnote on p. 176, it is seen at once that the aberrations of zero order are not independent, but satisfy the relation

$$(\Delta_0, \Delta_1, \Delta_2, \Delta_3, \Delta_4)(1 - \alpha_2 G, -1 + \alpha_2 S)^2\,(\alpha_1 - G, S - \alpha_1)^2 = W\,(1 - \alpha_1\alpha_2)\,(S - G)^4.$$

* "The Distribution of Corrective Duties in Optical Instruments," *Trans. Opt. Soc.* 24 (1922–23) 168.

This relation merely amounts to the statement that the spherical aberration of the stop centre is determined by the state of correction of the main image.

In consequence of this relation it is desirable to eliminate the stop aberration D_0 from the remaining equations by identifying one of the arbitrary magnifications with G. The equations giving the conditions for the separate components are then

$$\begin{aligned}
&\frac{1-G\alpha_2}{1-\alpha_1}(0,0,\tfrac{1}{2}+\tfrac{1}{3}\varpi_1,-\beta_1,\gamma_1\}2,0)^p(1+\alpha_1,1-\alpha_1)^{4-p}\\
&\quad+\frac{1-G\alpha_2}{1-\alpha_2}(0,0,\tfrac{1}{2}+\tfrac{1}{3}\varpi_2,-\beta_2,\gamma_2\}2,0)^{4-p}(1+\alpha_2,-1+\alpha_2)^p\\
&\quad-\frac{\alpha_1-G}{1-\alpha_1}(0,0,\tfrac{1}{2}+\tfrac{1}{3}\varpi_1,-\beta_1,\gamma_1\}2,0)^{p+1}(1+\alpha_1,1-\alpha_1)^{3-p}\\
&\quad-\frac{\alpha_1-G}{1-\alpha_2}(0,0,\tfrac{1}{2}+\tfrac{1}{3}\varpi_2,-\beta_2,\gamma_2\}2,0)^{3-p}(1+\alpha_2,-1+\alpha_2)^{p+1}\\
&\quad=\frac{4}{(S-G)^3}(\Delta_1,\Delta_2,\Delta_3,\Delta_4\}1-G\alpha_2,S\alpha_2-1)^p(\alpha_1-G,S-\alpha_1)^{3-p},
\end{aligned}$$

where p assumes the four values 0, 1, 2, 3. The various terms on the left side obviously refer to the aberrations of one of the components, with a stop in contact with it, for an object or image in coincidence with the other component. The equations are readily put into a form in which only one of these expressions appears. On the right S is of course a dummy and any value can be given to it; this is easily seen by assuming all aberrations to be removed for magnification G, when S occurs in the numerator only in the factor $(S-G)^3$ which cancels with the denominator. The four equations are then

$$\begin{aligned}
&(1-G\alpha_2)(1-\alpha_1)^2\{\beta_1-(2+\varpi_1)A_1\}+(\alpha_1-G)(2W+\alpha_1-\alpha_2)\\
&\qquad+\frac{2}{(S-G)^3}(D_1,D_2,D_3,D_4\}1-G\alpha_2,S\alpha_2-1)(\alpha_1-G,S-\alpha_1)^2=0,\\
&(\alpha_1-G)(1-\alpha_2)^2\{\beta_2+(2+\varpi_2)A_2\}+(1-G\alpha_2)(-2W+\alpha_1-\alpha_2)\\
&\qquad+\frac{2}{(S-G)^3}(D_1,D_2,D_3,D_4\}1-G\alpha_2,S\alpha_2-1)^2(\alpha_1-G,S-\alpha_1)=0,\\
&(1-G\alpha_2)(1-\alpha_1)^3\{\gamma_1-4\beta_1A_1+(3+2\varpi_1)A_1^2\}+2(\alpha_1-G)(1-\alpha_1)^2\{\beta_1-\varpi_1A_1\}\\
&\qquad-\frac{4}{(S-G)^3}(D_1,D_2,D_3,D_4\}\alpha_1-G,S-\alpha_1)^3=0,\\
&(\alpha_1-G)(1-\alpha_2)^3\{\gamma_2+4\beta_2A_2+(3+2\varpi_2)A_2^2\}-2(1-G\alpha_2)(1-\alpha_2)^2\{\beta_2+\varpi_2A_2\}\\
&\qquad+\frac{4}{(S-G)^3}(D_1,D_2,D_3,D_4\}1-G\alpha_2,S\alpha_2-1)^3=0,
\end{aligned}$$

where
$$A_1=\frac{1+\alpha_1}{1-\alpha_1},\quad A_2=\frac{1+\alpha_2}{1-\alpha_2}.$$

An interesting field in which to apply these equations is that in which the lens is corrected for aberrations in a primary plane for an infinitely distant object, so that the values to be inserted in the equations are

$$G=D_1=D_2=D_3=D_4=0.$$

For the purpose of a preliminary survey it is reasonable to put $\varpi_1 = \varpi_2 = 0{\cdot}7$, this being roughly equivalent to the assumption that each component thin lens will be achromatic and constructed of normal glasses of the older types. Denoting this common value for the moment by ω, the working equations can be put into the convenient form

$$\frac{\beta_1 + P_1}{3+\omega} = A_1 + \tfrac{1}{3}(P_1 - A_1)\left(1 + \frac{1-\alpha_2}{1-\alpha_1}\right),$$

$$\frac{\gamma_1 + P_1^2}{3+\omega} = 2A_1^2 + \tfrac{1}{3}(P_1 + 3A_1)(P_1 - A_1)\left(1 + \frac{1-\alpha_2}{1-\alpha_1}\right),$$

$$\frac{\beta_2 + P_2}{3+\omega} = -A_2 + \tfrac{1}{3}(P_2 + A_2)\left(1 + \frac{1-\alpha_1}{1-\alpha_2}\right),$$

$$\frac{\gamma_2 + P_2^2}{3+\omega} = 2A_2^2 + \tfrac{1}{3}(P_2 - 3A_2)(P_2 + A_2)\left(1 + \frac{1-\alpha_1}{1-\alpha_2}\right),$$

where

$$P_1 = A_1 - \frac{2(\alpha_1 - G)}{(1 - G\alpha_2)(1-\alpha_1)}, \quad P_2 = -A_2 + \frac{2(1 - G\alpha_2)}{(\alpha_1 - G)(1-\alpha_2)}.$$

The possibility of separating the factor $3 + \omega$ is to be expected, as the only condition imposed, besides those attainable in a single thin lens, is the flattening of the primary focal surface, the curvature of which in each component is represented by this factor. When $G = 0$, $P_1 = 1$, so that the first two equations are linear expressions for the determination of α_2 when α_1, β_1, γ_1 are given. The third and fourth equations become respectively linear and quadratic equations for P_2 on substituting for α_1, making

$$\left(P_2 + A_2\right)\left(1 + \frac{1-\alpha_1}{1-\alpha_2}\right) = \tfrac{1}{2}\{P_2(A_2 + 3) + A_2 - 1\}.$$

After P_2 is found, α_1 is derived from

$$\alpha_1 = \frac{1 + A_2}{P_2 + A_2}.$$

Alternatively P_2 may be assumed and A_2 found from the equations.

In this way diagrams may be constructed showing curves for which $\beta_1, \gamma_1, \beta_2, \gamma_2$ have constant values in the $\alpha_1\alpha_2$ plane. The values of β_1 and β_2 specify the shapes of the unit surfaces of the components, and γ_1 and γ_2 their spherical aberrations when used at magnification -1. The results represented in this way indicate how the aberrations of general thin components must be arranged to enable the curvature of the primary focal surface to be removed in addition to the corrections which are possible in a single thin system. For practical purposes the β_1 and β_2 curves give the shapes of the component lenses, and it is desirable to construct two other diagrams giving the values of

$$\gamma - (1 + 2\omega)\left(\frac{\beta}{1+\omega}\right)^2 = \Gamma, \text{ say,}$$

for the separate components as an indication of the kinds of glass which should be

employed if cemented components are to be used. Diagrams for the selection of suitable glasses when Γ is given have been previously described*.

It will suffice here to say that in a cemented doublet of glasses which give simultaneous correction for spherical aberration and coma for a distant object, as when a medium barium crown and a dense flint are used, Γ is approximately 0·35. As the refractive index of the crown glass falls Γ also falls. With a hard crown and dense flint such values as -5 to -10 are obtainable. With a soft crown or light flint Γ rapidly tends to large negative values. On the other hand large positive values for Γ are obtained if the index of the crown component exceeds that of the flint, extreme values with given indices occurring when the powers of the components are large in comparison with that of the combined lens. To use terms frequently employed in discussions of the principles governing the construction of anastigmatic lenses, the "old" achromats are characterized by small positive or by negative values of Γ, the "new" achromats by large positive values.

Two regions of the charts are of special interest, that in which both α_1 and α_2 are less than unity, the system consisting of two separated positive lenses, and that in which α_1 is less and α_2 greater than unity, the system being of the telephoto type. In the former region the Petzval condition cannot be satisfied, and the field under the conditions assumed may not be corrected in the most advantageous manner. It may however be noted that it corresponds to the type of correction aimed at by Mr Dennis Taylor in a wide angle anastigmat lens. With lenses of this type, when the separation tends to be small, at least one component inclines to an extreme form. In general the front component should be an old achromat, and the tendency on the whole is for it to assume a form convex to the incident light. There is, however, a region where the front component is of distinctly less power than the back, in which the front lens is decidedly concave to the incident light. In this region the back lens exhibits a less pronounced tendency to concavity to the oncoming light. These cases, as it happens, are not of practical importance as the curvatures of the surfaces are greater than with other constructions. Of greater significance is the fact that with components of about equal power the curvatures will be great unless the separation is at least a quarter of the focal length of the single component; both components should present their more convex aspect to the incident light, and the convexity of the front lens should exceed that of the back unless the separation of these lenses is greater than half of their focal lengths. A result with which modern practice is in conflict is that the back component, like the front, should be an old achromat. We are thus faced with the alternatives that either modern practice is wrong or that unsuitable conditions have been laid down. It is difficult to believe that the theory of two thin objectives is inapplicable to many types of photographic objective though it will be observed that this theory implies comparative neglect of the Petzval condition. The subject requires much fuller examination than is possible in a broad survey of the kind here attempted before the reasons for this conflict between theory and practice can be fully understood.

* *Trans. Opt. Soc.* 19 (1918) 169.

In passing it may be remarked that Rudolph's principle of the opposed signs of cemented surfaces in many cases amounts merely to the earlier rule of combining a new and an old achromat. On general considerations it is to be anticipated that Rudolph's principle can have no strict mathematical foundation affecting aberrations of the first order, for the same β and γ for any thin lens are attainable with two lens shapes by reversing the lens, the power of the cemented surface tending in the one case towards positive, in the other towards negative values. Moreover, in some of the best corrected lenses it appears difficult to find features consistent with any reasonable generalization of Rudolph's principle. On these grounds it seems permissible to offer the suggestion that it should rank as a guide to the designer in his first choice of systems for detailed consideration, the systems which it recommends being advantageous for manufacturing rather than for mathematical reasons.

The class of telephoto objectives must now receive some little attention, particularly in the region in which the Petzval condition is approximately satisfied. There is here less room for doubt than in the previous case that the conditions imposed approximate with fair precision to those which a real system should satisfy. Probably the insistence on the correction for distortion represents the most doubtful factor in comparing these theoretical results with existing constructions. The diagrams indicate that the front lens should be decidedly convex to the incident light, and should be constructed from the ordinary silicate glasses, thus being an old achromat. On the other hand, remembering that the rear lens is of negative sign, the chart giving β_2 shows that this lens must be concave to the incident light, the curves being decidedly bold. The Γ for this lens is also in all cases a large negative number, so that it must be composed of the older glasses with a limited difference between their dispersions. Thus here as in the previous case the barium glasses used for new achromats appear to be out of place in cemented components.

Other charts of interest besides those used for the purposes of the foregoing discussion may be constructed with advantage. Chief among these are those which show the changes necessary in β_1, γ_1, β_2, γ_2 when each of the aberrations D_1, D_2, D_3, D_4 in turn is made equal to unity. Since the equations are linear in these variables such a set of charts is sufficient to enable the necessary changes in the construction of the system to be read off directly, whatever aberrational conditions are imposed. As a rule it is simplest to construct these for $S = 1$, the reduction of any conditions to this form being carried out by means of the transformation equations given in an earlier paper*.

* "The Changes in Aberrations when the Object and Stop are moved," *Trans. Opt. Soc.* 23 (1921–22) 311.

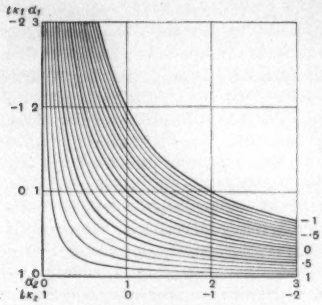

Fig. 1. Curves for constant values of $tk_{1,2}$.

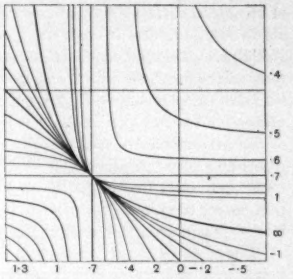

Fig. 2. Curves for constant values of ϖ.

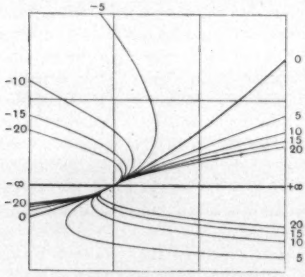

Fig. 3. Curves for constant values of β_1.

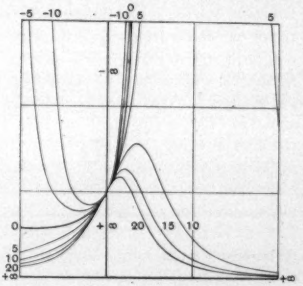

Fig. 4. Curves for constant values of β_2.

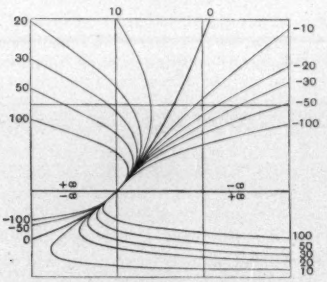

Fig. 5. Curves for constant values of γ_1.

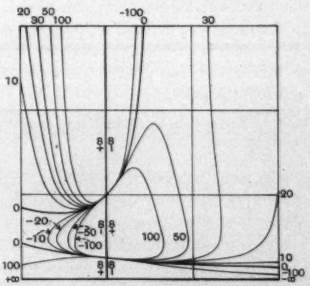

Fig. 6. Curves for constant values of γ_2.

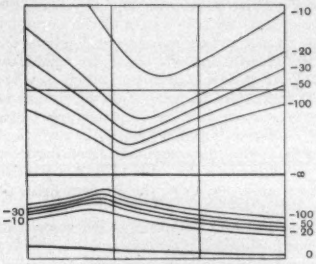

Fig. 7. Curves for constant values of Γ_1.

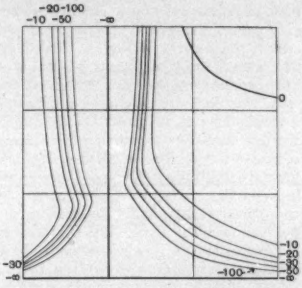

Fig. 8. Curves for constant values of Γ_2.

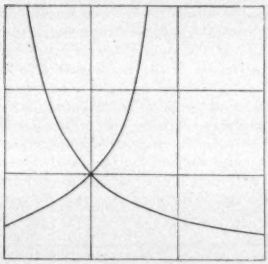

Fig. 9. Curves for $\Gamma_1 - \Gamma_2 = 0$.

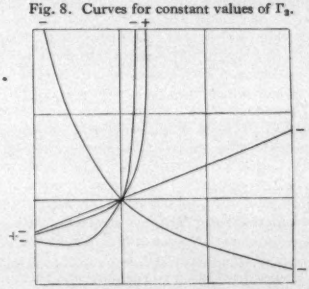

Fig. 10. Curves for $\beta_1 - \beta_2 = 0$ marked −
Curves for $\beta_1 + \beta_2 = 0$ marked +

Explanation of the Figures.

Each point on the diagram represents a lens of definite Gaussian construction, the scales of co-ordinates from which the construction may be read off being given on the left side and bottom of Fig. 1. For clearness these co-ordinates have been omitted from the other figures. The reference frame is the same for all the diagrams.

Photographic lenses composed of two converging components are represented by points lying within the bottom left-hand square of each diagram, both α_1 and α_2 lying within the limits 0 and 1. If the components are of equal power, the lens is represented by a point on the diagonal $\alpha_1 = \alpha_2$, those members in which the separation tends to be small compared with the focal length lying nearer to $\alpha_1 = \alpha_2 = 1$, those in which it is large nearer to $\alpha_1 = \alpha_2 = 0$. Lenses represented by points lying above and to the left of this diagonal have the front component of smaller power than the back, while the contrary is the case when the points lie below and to the right of the diagonal. Points in the four upper right-hand squares have both components negative, and the combination is also negative. The remaining two upper squares on the left contain points representing lenses with the front component negative and the back positive, while points in the central and right-hand squares of the bottom row have the front component positive and the

back negative, and thus include telephoto lenses. Of the last-mentioned four squares, only part of the area represents combinations of positive power, the boundary being the central thick curve of Fig. 1. This boundary appears again in Fig. 2 as the curve for which the Petzval coefficient is infinitely great. In this and the subsequent diagrams the lines representing zero and infinite values of the quantity depicted are shown by thicker lines than those representing other values. The coefficient β_1 plotted in Fig. 3, when multiplied by κ_1, the power of the front component, gives the sum of the curvatures of the unit surfaces necessary in this component to produce an image in the complete lens corrected for an object at infinity. Fig. 4 shows corresponding curves for β_2, the coefficient for the second lens. These values are realized by making the component lenses more or less convex or concave to the incident light, an increase in the value of β in a positive component being secured by making its surfaces more convex to the incident light. If the component is reversed, the value of β changes to $-\beta$.

Figs. 5 and 6 give respectively for the front and back components the amount of spherical aberration necessary when the component is used at magnification -1, positive values representing under-correction. Portions of some of these curves are omitted to avoid confusion where their general position is obvious. From Figs. 3 to 6 are derived the values of Γ_1 and Γ_2 shown in Figs. 7 and 8, these being constants, independent of the precise shape of the lens, which serve to show from what glasses the components should be constructed. Fig. 9 shows the curves $\Gamma_1 = \Gamma_2$, that is, the locus of points representing systems which may be constructed from cemented lenses of the same glasses in each component. One branch corresponds to lenses of very small power, and the other shows that in a system of this type built from two positive components the back should be more powerful than the front. In the regions between the curves which are shaped like the bowl and base of a wine glass Γ_1 exceeds Γ_2, showing that here the crown glass of higher refractive index should be in the front component if both are achromatized by the use of the same flint glass. Fig. 10 shows the curves $\beta_1 + \beta_2 = 0$, corresponding to the oppositely curved symmetrical and hemi-symmetrical lenses so largely used in the past, and also the curves $\beta_1 - \beta_2 = 0$, corresponding to constructions which have rarely, if ever, been employed. In considering this and any other β diagram it must be remembered that the shape is specified in relation to the power of the lens, so that a positive β corresponds in the case of a positive lens to the presentation of a convex aspect to the incident light, and with a negative lens a concave aspect. It may be inferred from these curves that in objectives approaching the symmetrical type the front component should generally be rather more pronounced in its curvature than the back.

By taking these charts in combination we see that it is possible to produce a corrected system having two similar cemented components made from the same glasses of approximately the shape required for the correction of coma in the single component with an object at infinity. The approximate position is given by

$$\frac{\kappa_1}{5} = \frac{\kappa_2}{8} = \frac{\kappa_{1,2}}{9} = \frac{1}{10t}$$

and these values suggest the use of the system in the reverse direction as a microscope objective rather than as a photographic lens. There is no other useful solution having these properties. This example illustrates the way in which such a set of diagrams, drawn on tracing paper and superposed, can be made to yield immediately, without any special calculations, the complete series of approximate solutions of any optical problem involving the removal of first order aberrations from a system of two separated lenses.

NEW TYPES OF LEVELLING INSTRUMENTS USING REVERSIBLE BUBBLES

By T. F. CONNOLLY, M.Sc.,
Inspector of Scientific Supplies, India Store Department.

MS. received, 20th September, 1923. *Read and discussed, 11th October*, 1923.

ABSTRACT. The paper describes the properties of reversible bubbles which can be used both graduated and ungraduated. Making use of certain of such properties, it can be shown how a true level can be obtained as the mean of two observations from a single station in an extremely simple manner.

The theory is then applied to the design of various types of "self-adjusting" levels. A series of actual levels, following the author's theory and specially made for the paper, are illustrated.

A specification for an ideal level, using the same theory, is given.

The application of a reversible bubble to the adjustment of a collimator is also dealt with.

It is thought that it may be of value to call attention to some uses of a reversible bubble, especially in connection with the adjustments of levelling instruments, theodolites and collimators. An accurate reversible bubble secures certain unique advantages in the determination of "true level" but its employment seems to have been restricted for no very good reason.

Properties of the Theoretical Bubble.

Theoretically in its simplest form the bubble may be ungraduated and consist essentially of an internally barrel-shaped glass container which holds a suitable liquid and encloses an "air bell." The sections of the surfaces are arcs of circles and they are symmetrical with respect to one another and to an intermediate line which may be called the axis*. In any gravitational field, with the bubble axis approximately horizontal, the "air bell" takes up a definite position in relation to the upper curved surface of the tube. It is clear that rotation of the bubble about its axis of symmetry (or an axis parallel to it), will not cause displacement of the "air bell" whatever its position may be (Fig. 1). It follows, therefore, that by making use of such a property the direction of this axis may be found or, as is usual in practice, a mechanical axis parallel to it can be determined. No information, however, is given by this as to the tilt of the axes concerned relative to the gravita-

* The axis of the bubble may be defined as the line which lies midway between the upper and lower curved surfaces and at right angles to the line joining the centres of curvature of the surface sections.

tional field. If, however, the bubble tube is calibrated for the level position (represented by arrows in Fig. 2) and the rotation round the bubble axis is made with the "air bell" in this level position a critical condition is established. Consequently, if in the prescribed conditions the "air bell" is not displaced relative to the observer, the axis of rotation must be truly level. If the mechanical axis include an optical system which is in collimation, a true level sight is obtained. In this case all three axes, viz. bubble, mechanical and collimation, are either parallel or coincident and all of them are truly level. So far as is known an instrument possessing the practical equivalent of the above properties has never had its properties fully exploited.

Fig. 1.

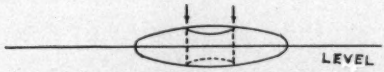

Fig. 2.

The Reversible Bubble of the Zeiss Level.

An outstanding practical application of a bubble somewhat similar to the above occurs in that remarkable instrument, the Zeiss level. Here the partial accuracy of the bubble plays a very subtle part which is either overlooked or taken for granted in the literature of the subject. True level is obtained with the Zeiss instrument as the mean of four separate observations. The result is remarkably independent of imperfections either in the bubble reading system or in the bubble itself.

When, however, the length of the "air bell" used for the initial setting varies, as it will in the course of work owing to changes of temperature, coincidences in the prism reader will not be preserved unless the change is symmetrical with respect to the centre of the "air bell." This means that each worked surface of the bubble must be of uniform curvature.

It is remarkable that such uniformity is the only element of geometrical accuracy which is essential in normal practical use. In fact, if the Zeiss reversible bubble be looked upon as two separate bubbles, it is not essential that they be of the same sensitivity; moreover, their "axes" may make any angle with one another, either in the horizontal or vertical planes. This latitude is of course of practical value in simplifying manufacture, but it necessitates *four* independent observations, the calculation of a mean, and an instrument setting to the calculated value, before a truly level instrument is obtained.

Reverting now to the first mentioned ideal bubble in association with mechanical and optical axes, it will be seen that the practical equivalent of the theory would be

the use of a telescopic system with adjustable graticule, rotating in bearings, to which a bubble as described is attached. Calibration of the bubble for the true level position could be made, in conjunction with a standard collimator, by using a workshop fixture equivalent to the mechanical and optical system which will finally carry the bubble. It will be obvious that this practical embodiment of the theory represents a "self-adjusting" level. Its accuracy will depend on how closely theory can be carried out in practice.

The Requirements of a Practical Bubble.

So far as is known, no instrument has ever been constructed to utilize fully the symmetry and co-ordinated accuracy of the theoretical bubble above described. It is indeed not clear whether any attempt has ever been made previously to produce a reversible bubble with a double-sided symmetry of high accuracy, though in the catalogue of A. Pessler and Son, Saxony, reversible bubbles are described as having "two equal parts *exactly opposite* to each other." These bubbles are catalogued with sensitivities down to 2 seconds of arc per millimetre.

In point of fact reversible bubbles as used in the Zeiss level, as made by both Zeiss and Cooke, Troughton and Simms, have been tested and found to be of the full symmetrical accuracy necessary for the level described by the author. This high quality is in these cases gratuitous, for it is not *essential* to the theory of the adjustments of the instrument. Even if the upper and lower curved surfaces be slightly decentered with respect to one another, a centre can always be found (unless the defect is extreme) and the equivalent of a perfect bubble realized. The problem is similar to that involving decentered surfaces in a lens.

For the sake of setting a specific concrete problem the author formulates a specification for a *high grade* reversible bubble and hopes that its commercial production will be tackled generally. The sensitivity of the bubble required would be about 5 seconds of arc per millimetre. Both the accuracy of the bubble "run" and the symmetry should be as high as possible and should be certainly within $\pm$ 1·0 second of arc. The bubble should be so calibrated that, with its axis horizontal, it indicates true level. Reversible bubbles of lower grade can of course be produced more easily and in fact are now commercial propositions with a number of firms.

The Theoretical Level.

The ideal self-adjusting level aimed at by the author would have its practical counterpart in an instrument having the fully symmetrical graduated bubble specified above and adapted for use with a practically perfect Zeiss prism reading system. As already pointed out, the Zeiss level bubble is *ungraduated* and for quite good reasons. The author suggests that if any such bubble, which in instruments as made by either Zeiss or Cooke, Troughton and Simms is already of sufficient accuracy, be assumed to be quite symmetrical and be calibrated for the level position on one side only, the level without further change could be used in a new way. The prism device is first adjusted to give coincidence of the bubble *gradua-*

tions for the two ends. The bubble is then adjusted wth respect to the telescope so that on rotation there is no alteration in a *bubble* coincidence setting. When this is done, with a telescope in collimation, a true level is found.

This method of obtaining a true horizontal direction depends only on instrument constants and involves, outside the instrument, no apparatus of any kind. Any fixed object in the field of view will serve to adjust telescope collimation. Diaphragm adjusting screws and full bubble adjustments are essential. Once the

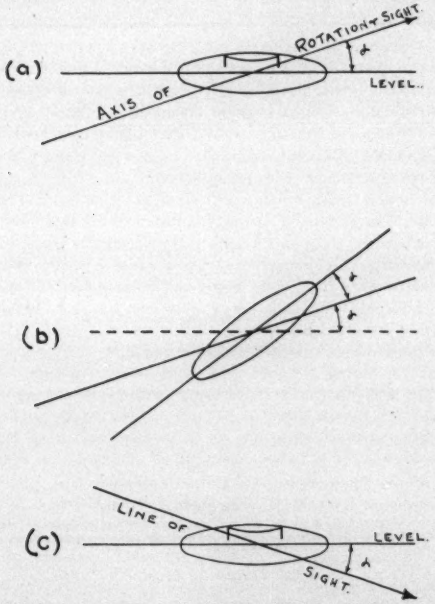

Fig. 3.

adjustment is made, however, it can be checked by the simple operation of a semi-rotation. It will be seen that such a level is a complete embodiment of the theory first set out and that, when in adjustment, with all axes level, it corresponds to Fig. 2.

With the ideal bubble as shown, and still using a high grade Zeiss prism reading device, level can be obtained also as the mean of two observations on a single graduated staff. The telescope is first approximately levelled. The prism device is

then adjusted to the graduations and the telescope tilted by the gradienter screw until bubble coincidence is obtained (Fig. 3 (*a*)). A staff reading of the telescope setting is then made. The usual semi-rotation is given to the telescope and bubble system (Fig. 3 (*b*)), and a bubble coincidence is again secured (Fig. 3 (*c*)). A second staff reading follows. The mean reading represents true level. The bubble is then set to coincidence, with the telescope aligned on this mean reading and an adjusted instrument is secured.

It is of considerable practical importance to note that, for a level as specified and used as described, exact collimation of the telescope and perfect adjustment of the bubble with respect to the telescope are not essential. It would indeed seem possible to eliminate collimation adjustment and to make the bubble a fixture with respect to the telescope, the only condition being that the range of movement of the prism reader must be sufficient to take up imperfections in collimation and bubble telescope alignment. Such an instrument should attain a very high standard of stability and permanence in adjustment.

Though the elimination of adjustments might be aimed at, probably in practice it would be found necessary to supply them. They should, however, be made so solid and should be so protected that they would only be needed after misuse or damage to the instrument. It will be clear that the practical instrument at which the author is aiming would be extremely robust and would yet possess the property of being "self-adjusting" from a single station with a minimum of trouble.

A levelling instrument with a reversible bubble as specified and a prism reading system of equivalent accuracy, used according to the preceding theory, is considerably more simple in use than the present Zeiss level and yet is designed to do everything of which a Zeiss level is capable. On account of its simplicity it could be adjusted more quickly and should be more solid and lasting. The necessity of end to end reversal is eliminated and the extra objective, essential in the Zeiss telescope, is not required.

A Practical Application of the Theory.

For a practical level of lower grade less accurate elements could be used. The mechanical and optical parts of the specification for the present "Precise" level of the India Store Department could be employed. A reversible bubble having a sensitivity represented by 40 seconds of arc per tenth inch and reversing with such accuracy as to be within the thickness of the etched graduations (approximately $\pm$ 2 seconds of arc) is obtainable commercially. The combination of these elements with a well-made Zeiss prism reader represents a practical embodiment of the theory. Such a specification represents fair co-ordinated accuracy. The accuracy of setting of the bubble attained by means of a Zeiss prism reader greatly exceeds that of a 20-second bubble read in the ordinary way or with the bubble ends read against the graduations. The error of setting may be taken as being from 1 to 2 seconds of arc. This, though of the same order of magnitude, exceeds the resolving power of the telescope (3 seconds of arc approximately) in conjunction with which the bubble is employed. The accuracy of reversal of the bubble, which is such as to

be within the thickness of the etched graduation (say ± 2 seconds of arc) completes the co-ordination. Fig. 4 represents such a level which has been constructed for the author by Messrs E. R. Watts.

It will be noted that variations due to temperature, *which do not disturb bubble coincidence in a prism reader*, have been neglected in the whole of the above discussion. In order to neglect temperature in this way it is essential to have *uniform* curvature for each bubble surface. This condition is essential, as in the case of the

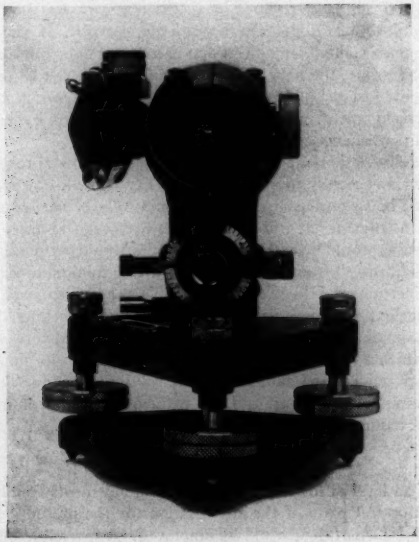

Fig. 4

Zeiss level, and indeed applies to all bubbles in practical use whose length varies with temperature and to all bubbles without exception when used for difference readings. The condition, however, is not a stringent one as any decent commercial bubble meets the requirements.

Levels made to a Simplified Theory.

It is not, however, always necessary to insist on the condition of symmetry for certain types of instruments. If a bubble lacking this accuracy is *independently* calibrated on both surfaces it is capable of reproducing the level position when the

bubble ends remain in coincidence with the related fiducial marks after rotation into positions 180° apart. Messrs W. F. Stanley appear to have broken ground with a very interesting level depending on this fact. In this level, which in a modified form is described later, it will be noted that, though performing similar functions, the bubble has not the complete accuracy of the ideal bubble first mentioned and should be distinguished clearly from it.

After discussing an early draft of the present paper with the author, Messrs E. R. Watts have constructed a level (Fig. 5) on this principle of independent calibration, but using their patented "Constant" bubble. Owing to the constancy of the bubble

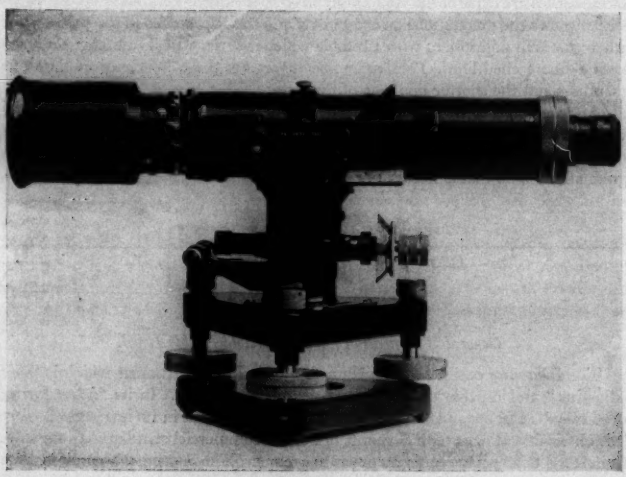

Fig. 5.

length one end only of the bubble need be read and pointers indicating bubble ends 180° apart are sufficient for adjustment and setting. The constancy of the end setting also allows of a very convenient method of prism reading and this has been adopted by the firm.

We can now consider the use of a reversible bubble, each side of which is independently graduated, *in conjunction with a Zeiss prism reading system of average accuracy*. The method employed in obtaining true level does not differ greatly from that already described for an ideal bubble graduated on one side only. The prism reader is first set to the upper bubble graduations. A bubble coincidence is then made and a staff observation obtained. After rotation the prism reader is then re-set to the lower graduations. Coincidence setting for bubble then follows and a second

staff observation is secured. A true level is obtained as a mean of the staff observations, exactly as before. A "practical" level as outlined above obviously approximates more and more closely to the "ideal" as the standard of manufacture is raised.

The method of marking the bubble, devised and recommended by the author for use with the Zeiss prism reader, applies to any reversible bubble, whether it be perfectly symmetrical or not. The upper surface is marked with two long fine lines indicating bubble ends. The under surface has two similar marks but shorter and set inwards as though for the ends of a shorter "air bell." The prism reader can be set for coincidence of either pair. If the bubble and prism system are accurate, both coincidences are made at the same time and so re-setting on rotation is not necessary. Fig. 6 shows the graduations on the bubble and the appearance in the prism reader when this is in adjustment with a bubble which is symmetrical to within the thickness of the etched lines. The larger line represents the coincidence of the upper markings and the shorter line that of the lower.

After experience with several levels, the author does not recommend a bubble which requires the prism reader to be re-set on rotation. The necessary symmetry in the bubble can be obtained by good work. In this case one setting against graduations is all that is necessary. This corresponds exactly to the early theory.

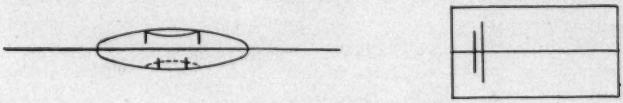

Fig. 6.

General Specification for a Reversible Bubble Level.

The following outline specification for a levelling instrument which is "self-adjusting" and capable of the highest accuracy is offered. In its *highest* form it demands the bubble and prism system as formulated, but in its less accurate form it is obtainable at once with elements of normal commercial standard. In the latter form it meets all usual requirements as to accuracy. The instrument would include:

(1) A large aperture internal focussing telescope without reversible optics;
  (*a*) mounted so as to be capable of a half rotation and
  (*b*) tilted by a micrometer differential screw, or equivalent device, for fine subtense readings.

(2) A certified reversible bubble graduated on one or both sides of either the "constant" or ordinary type.

(3) A prism device with movable prism system, the present Zeiss device or its equivalent, for reading the bubble.

(4) An ordinary tribranch and toe screw, or some ball and socket quick setting system, for approximate adjustment checked by a "T" or "L" bubble.

Where the highest accuracy is aimed at, the parts should be in steel.

The above combination of elements is believed to be quite new and the resulting level should prove to be of universal value.

Some Specially Produced Levels.

So far the paper has been concerned with theory and with suggestions for practical levels. The matter has not been left in this stage. All the chief makers of surveying instruments in this country have been interested in the matter and various types of levels using graduated reversible bubbles have been constructed. In every case the instruments have been made specially for this present paper.

Four distinct practical types have been evolved. In all of them it has been found desirable to graduate independently both sides of the bubble, using a workshop fixture which gives rotation through a standard interval of 180°. When a suitable and accurate fixture is employed, such calibration may be taken as being a factory routine job.

The optical and mechanical design of all types has been based on the existing specification for a "Precise" level as used by the India Store Department. This represents an internal focussing level of eleven inches equivalent focal length, with an aperture of 1·65 inches and a magnification of between 28 and 32 diameters. The gradienter device employed is a differential screw, or its equivalent, giving for one complete turn of the drum an interval of one part in one thousand. With a drum divided into 50 parts numbered as 100 the unit reading will therefore be one part in 100,000 or approximately 2 seconds of arc.

It has often been stated that the ideal level should aim merely at simplicity and sturdiness. It is argued that it is preferable to achieve the permanent retention of adjustments rather than easy adjustability. I concur in large measure with this opinion. The India Store Department specification, which has been adopted as the basis for the new level, was drawn up, in fact, from this point of view.

A further practical question will then arise as to what sacrifice of simplicity is permissible or what other sacrifice may be made in order to secure a "self-adjusting" instrument.

In every level made to illustrate this paper the original simplicity and sturdiness of the telescope and bubble system have been retained unimpaired. Efforts indeed have been made to *increase* them. The new levels, therefore, are no less sturdy and stable than their predecessors. The sole essential change made is in the provision of a bearing on which the telescope and bubble system can rotate. It will be realized that the addition causes little interference with retention of adjustment and it is not considered too high a price to pay for the ease of adjustment which it secures.

This mechanical change from the non-adjustable type, in conjunction with the use of a reversible bubble needing *two* observations only, has resulted in the production of a "self-adjusting" level which is the simplest so far obtained.

It is hoped that levels of this type may have a successful future.

Levels by Messrs W. Ottway and Co., Ltd.

The most straightforward embodiment of the principle is shown in the level produced by Messrs W. Ottway and Co., Ltd. (Fig. 7). The bubble, which is independently graduated on two sides, is read by a mirror reflecting device in the

usual way. When, with a telescope in collimation, the bubble remains in the centre of its run, after rotation, adjustment has been secured. Variations of temperature do not interfere with a bubble setting.

Several factors militate against work of high precision with such a method of reading. The position of each extremity of the bubble is *estimated* only as to its position between neighbouring graduations. The position of the opposite end is similarly obtained and an eye balance is struck to obtain a setting. This apparently simple operation involves, therefore, two estimations and a final balance, this last necessitating a movement of the eye from end to end of the bubble.

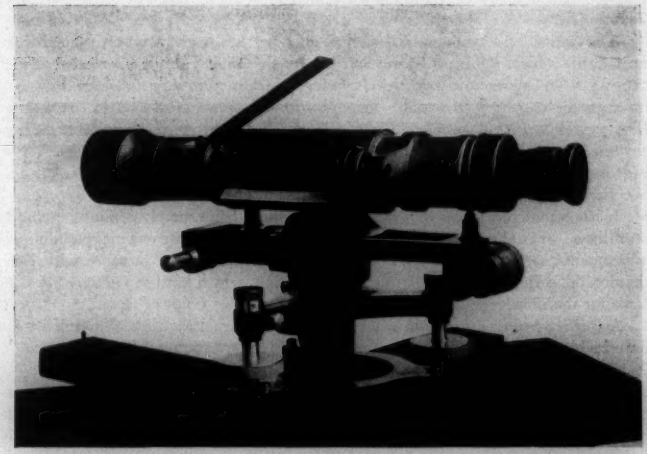

Fig. 7.

A further error may arise from the fact that the "air bell" ends and the graduations are separated in plane by the thickness of the glass. There is therefore the possibility of parallax for each end. Also from any one position of viewing, a differential parallax as between the end readings is not merely possible but is inevitable. It is clear, therefore, that there are many sources of small errors and a mirror reflector is not recommended for a level of great accuracy.

In spite of these defects, which apply to the vast majority of levels as actually made, such an instrument meets average needs in a simple way.

A detail of some interest in connection with this instrument is the use of steel for the telescope body and bearing. There can be no question that this change from

current British practice allows of a neater and stronger mechanical design. With careful finish the risk of corrosion, either external or internal, is negligible.

The use of steel has, however, what may be considered the defect of preventing the use of a magnetic compass as an adjunct to the level. From the author's point of view such an accessory could be dispensed with without regret, its place being taken satisfactorily in nearly every case by a graduated circle. Any magnetic work necessary could be done by means of an independent prismatic compass with considerably higher accuracy than is possible with the small compasses usually attached to levels.

The accuracy of a level compass when it is new will certainly not exceed $\pm \frac{1}{2}°$, which is about one part in 120. The resolving power of the level telescope, the unit gradienter reading, and the accuracy with which the bubble can be read are all of the order of one part in 100,000. The conjunction of an accessory which is sensitive to one in 120 with an instrument of such refinement always strikes the author as the *reductio ad absurdum* of co-ordinated accuracy.

The level could, however, be made, if required, in non-magnetic material and thus permit of the use of a compass.

Levels by Messrs Cooke, Troughton and Simms, Ltd.

The level produced by this firm is a modification of their well-known precise level which is to India Store Department specification. The rotation is a single collar which is dust and moisture proof. The bubble reading is by the firm's patent speculum mirror. This device has the advantage of reducing the apparent size of the bubble, thus minimizing the eye movement necessary for estimation, and it reduces differential parallax to a negligible amount. The instrument is shown in Fig. 8.

This firm has also graduated the bubble in one of their Zeiss pattern levels. The bubble is of high symmetry and, used in the level according to the author's theory, gives, with great accuracy, the results expected.

Level by Messrs E. R. Watts and Son, Ltd.

The next level incorporates essentially the patented "Constant" bubble of Messrs E. R. Watts. Because of its property of not changing length with temperature one end only of the "air bell" is set at a fiducial mark. It is obviously not essential to trouble about the opposite end. Bubble reading, as compared with a simple mirror reflector system, is therefore simplified in two ways, viz. (*a*) one end only needs attention, and (*b*) the setting is to a line and not by estimation of position between marks. The difficulty as to parallax in the observations is not, however, avoided. In the use of the level alternate ends of the reversible bubble are read by means of prism reflectors (Fig. 5). Each setting is made by bringing the "air bell" ends up to the middle fiducial lines. The construction of the instrument is neat and the observations are very conveniently made.

The setting to a line, though apparently a very simple operation, is found to give rise to differences in observation with different observers. Settings may be "fine"

or "full" in various degrees depending on the individual, who is presumably aiming at a tangent setting.

It is of interest to note in passing that if the bubble length were subject to a secular change the determination of level would not be affected, the errors of the two essential fundamental observations being equal and opposite in sign.

In both this level and those previously described the bubble when being adjusted is pulled to the "level" position by means of the usual antagonizing screws. Such screws are found to be very troublesome in the field. The adjustment after a short

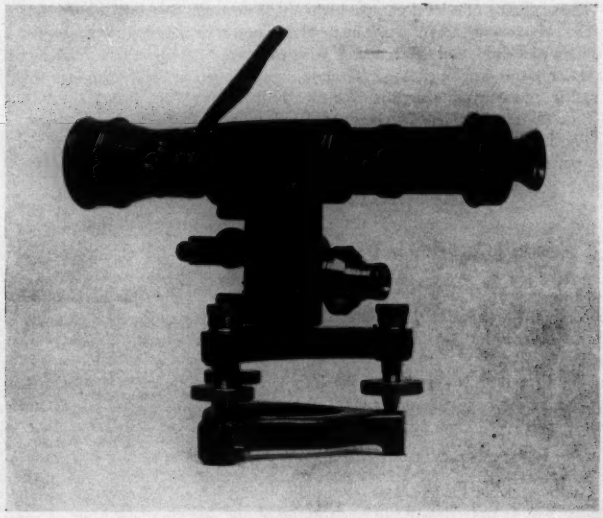

Fig. 8.

time is found to be changed, due perhaps to strain and elastic give in the metal. It is necessary in practice to operate with extreme care and for safety to check after the lapse of an hour or so. This defect is overcome in the author's level described later.

As a minor point of design, attention is called to the fact that in this level the usual circular bubble for rough setting up has been replaced, at the author's suggestion, by two small bubbles at right angles to one another. This appears to give more convenient and quicker setting when using the ordinary three-screw adjustment.

Level by Messrs W. F. Stanley and Co., Ltd.

The level due to Messrs W. F. Stanley and Co. introduces a new patent method of reading the bubble, which is similar to the Zeiss prism device. A side view of the level is seen in Fig. 9. Here again the bubble is graduated independently on both sides, while the reading is made by two fixed prism reading systems. Attention is called to the compass and its optical reading device (not shown in the figure).

A further level, embodying two fixed plain mirror reflectors, has also been constructed by the firm. This is a neat and straightforward instrument.

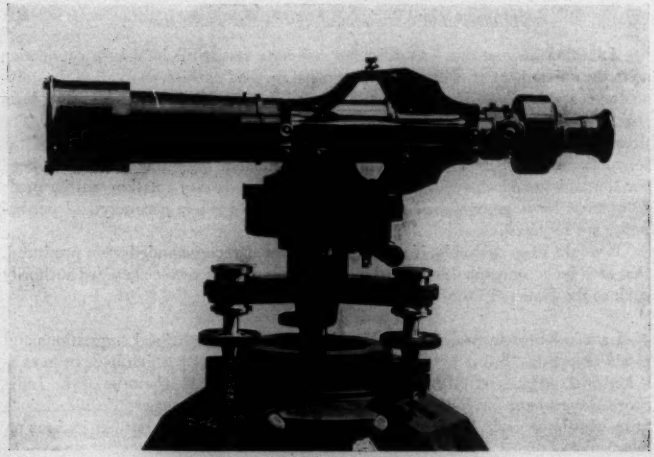

Fig. 9.

The Author's Level with Zeiss Prism Reader.

The last level embodies the author's original suggestions. It consists essentially of a standard India Store Department level with the Zeiss patent prism reading device used in conjunction with a reversible bubble as already described.

This prism reading system is considered to be quite definitely the best method of bubble reading yet devised. It practically eliminates parallax and "personal equation" from bubble setting. As previously stated in this paper, the estimated accuracy of setting is from three to four times as great as that for a bubble of equal sensitivity read from one end only.

After obtaining level, the final adjustment is made by sliding the prism box. This is a smooth operation and involves no elastic metal strain such as is usual

with adjusting screws. The bubble antagonizing screws, though supplied, are for rough preliminary adjustment only and would rarely be required with a level as designed.

Once the difficulties of manufacture of the bubble and prism system are overcome and their accuracy tested and certified, the user has nothing to worry over. The setting of the bubble is easy and certain and the operations are of extreme simplicity.

The level, made by Messrs E. R. Watts, is shown in Fig. 4. A provisional patent for the instrument has been taken out by the author.

Adjustment of a Collimator with a Reversible Bubble.

Attention may be called to a further use for a reversible bubble in connection with the adjustment of collimators. The usual method consists in treating the collimator as a Y level. With rotation and reversal in open Y's in the usual way a level can be found either with or without the use of an auxiliary striding level. The accuracy, however, depends essentially on the equality of the diameters of the telescope bearings. By the use of the fully graduated reversible bubble the need for the end-to-end change is eliminated and the necessary rotation can be made in covered bearings, which have obvious advantages where accuracy and permanence are required.

It would seem desirable, if reversible bubbles as recommended were produced, that each be accompanied by a certificate of accuracy from some standard authority such as the National Physical Laboratory.

I am indebted to Major E. O. Henrici, R.E., for very helpful suggestions and to his papers on "Spirit Levels"* and "Precise Levelling"† which have been consulted with advantage. My thanks are also due to Mr E. H. Coumbe of the India Store Department for assistance rendered in preparing the paper. To the various firms who have made levels specially for the meeting I tender my felicitations on the results obtained, and I wish them success.

APPENDIX.

Effect of temperature on a Zeiss prism reader.

To ascertain the range of temperature through which the prism bubble-reader is usable, the instruments *A* and *B* were placed on a grid in an asbestos-lined oven, heated by electricity. Bubble settings were made, the temperatures noted, and the apparent position of coincidence in the field of view of the prism reader ascertained. A temporary scale 0 to 10 was used to define the positions of setting. The following results were obtained:

* *Trans. Opt. Soc.* 20 (1918–19) 45.
† *Proc. Inst. Civ. Eng.* vol. 209, part i.

Series	Level	Temperature F.	Position of setting
(1)	A	72	6·5
		80	7·5
		118	Out of field of view
(2)	A	75	5
		88	6
		115	10 (just in view)
(3)	B	75	3·5
		94	3·8
		114	5
		127	7
		140	9

The thermometer was kept in contact with the bubble of the instrument. The duration of the test was about one hour for each instrument, the rise of temperature being gradual throughout.

The upper temperature limit for the practical use of the better instrument is, therefore, about 140° F.

The instruments were also subjected to a cold test. By the kindness of the Union Cold Storage Co., use was made of one of their cold store-rooms. The instruments were set up in a *dry* cold room and in both cases the setting remained well in the field of view for temperatures about 13° F.

It will, therefore, be seen that a well-made Zeiss prism reader is independent of temperature changes within the limits of 13° and 140° F.

DISCUSSION.

Major E. O. Henrici: The author is to be congratulated on having proposed and worked out an application of the reversible bubble to the engineer's level, and showing that the accuracy of the bubble tubes commercially obtainable is sufficient for this purpose. The question whether the gain in the ease of adjustment compensates for the extra cost of the instrument as compared with a suitable dumpy level is one that can only be decided by the user according to the circumstances of his work.

In the most precise class of work extreme accuracy of adjustment is not necessary, the sights must for other reasons be of equal length, or nearly so, and the adjustment error cancels out. In this case it is, however, important to set the bubble with a high degree of constancy, i.e. the bubble must be sufficiently sensitive, and be easy to set accurately to its marks; for this the Zeiss prism device is the best I have come across.

I can, however, conceive of many cases where it would be of value to be sure that the mean of two readings is free from adjustment errors, and this can be done with most if not all of the instruments exhibited. I was particularly struck with the adaptation of the "constant" bubble to an instrument embodying the author's ideas.

With a mirror viewer for the bubble the user must be careful to avoid parallax

errors in setting the bubble; these errors may arise in all the instruments exhibited, except those using the Zeiss prism device, if the user does not always view the bubble from the same position. I agree with the author that the Zeiss level is unnecessarily complicated and therefore expensive.

Mr G. W. Watts: Reference has been made in Mr Connolly's useful paper to precise levels being fitted with a magnetic compass. As manufacturers, we have to supply one when ordered, which is fairly often, so we have designed an attachable one. When laying off horizontal angles it is obviously better to depend on a graduated circle and index, and levels fitted with this can be procured. A forgotten steel rule in the waistcoat pocket is liable to render compass readings unreliable.

As regards the Zeiss prism reading device for bubbles—there is no doubt that it is the easiest and also most accurate method, but there are one or two things that occur to me: I have an idea that when used in extremes of heat or cold the coincidence looked for in the top prism may have disappeared out of the field of view to the right or left respectively, unless the prisms are made sufficiently large; again, being a proprietary article of Messrs Zeiss, there are legal restrictions to its promiscuous manufacture except, of course, under license from the patentees; obviously also it is much more expensive to produce.

Other reflecting bubble reading systems have been said to possess parallax; this may be true but, if the instrument has been well designed, parallax is not very evident with an ordinary normal view of the bubble and it occurs to me that it can be completely remedied by marking—not etching—the reading line (a single one only at each end is necessary in the case of a "constant" bubble) on the *inside* of the glass bubble tube, as is sometimes done now in circular bubbles.

Mr S. Weston: I am very glad that Mr Connolly has discovered a method of levelling which promises to increase the utility of the spirit level, and may stimulate the efforts which are being made to further its accuracy.

It occurs to me that, in the case of bubbles used as proposed by Mr Connolly, tests of a very rigid and exhaustive character should be employed before the spirit level is finally mounted on the instrument. Such tests would, of course, increase the cost of production, but would seem necessary in view of the extra demands which would be made on the bubble.

Mr J. W. Atha: I wish to congratulate Mr Connolly upon the excellence of his paper, and at the same time correct his statement that all the levels exhibited were specially made for the reading of his paper. The Zeiss instrument, of which he has spoken so highly, is a No. 2 Standard Level and was not constructed specially, as perhaps may have been conveyed to the meeting in error. I wish, however, to thank him very cordially for his appreciative remarks, which I shall have pleasure in conveying to Messrs Zeiss, Jena.

Mr Connolly: I am glad to note that Major Henrici, whose exceptional experience and wide knowledge of the subject I cannot pretend to, is in general agreement with the views expressed.

The instruments are not on a production basis as yet and the extra cost and difficulty of manufacture, though demonstrably small, is likely to be over-estimated. I am hopeful that the "user" will not be unduly taxed for the extra convenience offered.

Mr Watts' remarks as to the temperature limits within which a Zeiss prism reader can be used are to the point. The temperature limits of use have already been investigated by myself. From the appendix it will be seen that the system was usable between limits of 140° F. and 13° F. (19 degrees of frost).

Mr Watts' suggestion as to the avoidance of parallax is interesting. If a suitable permanent mark on the *inside* of the bubble tube can be made, a very great improvement will have been effected so far as parallax is concerned. Its application will not be limited to a constant bubble. The same method of graduation applied to an ordinary bubble would be of high value if successful.

Mr Weston's criticisms are in effect quite sound, but I think the extra cost of the tests referred to has been exaggerated. Apart from a *single position* check on exact reversibility, the only other condition involved is one of uniform curvature for the ground bubble surfaces. With modern methods of production this last condition is fulfilled automatically, just as it is for spherical surfaces in lens work. Similarly in the case of a ball bearing of high precision the conditions are equality of diameters and perfection of sphericity. Yet we know that hardened steel balls of uniform diameter and amazingly perfect sphericity are produced commercially at "ten a penny." Commercial tests of the reversible bubble would, therefore, be reduced to *one* which checked the exact reversibility of the bubble.

Mr Atha has of course misunderstood my statement. All the levels shown, with the exception of the Zeiss level, kindly lent by Mr Atha, and one other which was specially modified, were constructed for the meeting.

Being engaged in the daring task of simplifying and therefore, in my opinion, improving the Zeiss level, I am glad to record my high opinion of its theoretical and practical merits.

EXHIBIT OF AN OPTICAL REVOLUTION-COUNTER

By B. K. JOHNSON

(Optical Engineering Department, Imperial College of Science and Technology.)

13th December, 1923.

It is well known that when a truncated right-angled prism is rotated about an axis parallel to the hypotenuse, an object seen through the prism appears to rotate in the same direction at twice the speed. In the instrument exhibited this principle has been utilized in order to enable the speed of a rapidly-revolving object, such as a turbine shaft, to be measured without the necessity of coupling the measuring device directly to the object. The instrument consists essentially of a rotating prism and a speed indicator geared to it. The speed of the prism is adjusted until, on looking through the prism, the rotating object appears to be stationary. The speed of the object can then be read off on the indicator. A full description of the instrument, including suggested modifications,-such as the use of two prisms revolving in opposite directions, has already been published*.

DISCUSSION.

Dr J. S. Anderson: Mr Johnson's application of the well-known principle of the rotating prism does not appear to be new. A few years ago Pritschow wrote a paper† describing various applications of rotating prisms or mirror systems. According to him an instrument built up on this principle was invented by Prof. M. Thury and called "le cyclostat," while Dr D. Thoma employed a similar instrument, called a "rotoscope" (manufactured by Messrs Voigtländer), for observing the stream lines in turbines. At the Royal Society Soirée in 1922 Dr E. H. Rayner exhibited an instrument‡ embodying the same principle. Such instruments appear to have been used not so much for the measurement of speeds as for the observation of fluctuations from the steady state.

Mr J. Guild: I would like to emphasize Dr Anderson's remarks. The device has been familiar to me since towards the end of the war when I heard it suggested as a means of examining rotating parts.

As regards the proposed use for measuring turbine speeds, I gather from Mr Johnson's remarks that the only advantage is due to the twofold, or possibly fourfold, reduction of speed, and the consequent diminution in wear and tear of the tachometer. Surely the obvious remedy here is to gear down the tachometer so that the latter is working at a suitable rate. Any desired speed reduction could be

* *Optician* 66 (7th Dec. 1923) 304.

† *Centr. Ztg. f. Opt. u. Mech.* (1st Feb. 1920).

‡ *Engineering* 113 (1922) 659.

employed without introducing an optical device with the consequent need of adjustment before a reading can be taken.

Mr Johnson: In replying to both Dr Anderson and Mr Guild I would like to point out that, although the principle of the rotating prism may not be new, its application for counting revolutions (as the title of the exhibit implies) appears to be of a novel character; and further the idea of using two such prisms mounted in optical train (and made to revolve in opposite directions) does not seem to have been previously conceived. Quite apart, however, from any so-called new discoveries, the carrying-out in practice of any principle is a very different matter to suggestions made on paper and it is with this point in view that the exhibit was made.

REVIEWS

Les Applications des Interférences Lumineuses. By CH. FABRY. Pp. 160; 58 Figs. (Paris: Published by the Revue d'Optique théorique et instrumentale, 1923.) Price, 10 fr.

It is doubtless in conscientious fulfilment of his duties as Directeur Général de l'Institut d'Optique Théorique et Appliquée that the author, so distinguished for many new and important applications of the interference of light, has written this book. It is the most complete, indeed the only even approximately complete, summary of such applications which the reviewer has yet seen. Michelson's *Light Waves and Their Uses*, published in 1902, is no longer an adequate presentation of the subject—largely owing to progress which his own work initiated, and the present book is therefore much needed. Though one may a little grudge the time Professor Fabry has diverted from original researches to its compilation, it is a book which no one but he or Professor Michelson could write with like authority.

The space devoted by the author to his own work is considerable, but that of Michelson and others is also adequately dealt with. The presentation has the clarity, logical progression, and completeness which one has learnt to expect from Professor Fabry.

Chapter I deals with the general nature of interference of light, and with the conditions and instruments for producing the phenomenon. The first part of Chapter II contains an account of applications to the measurement of small displacements or small deformations. The second part of Chapter II (pp. 79 to 95) is occupied with a condensed, but (having in mind the small space occupied) very complete account of the use of interference in metrology, and particularly to the determination of the metre in wave lengths of light by Michelson and by the author. The agreement between Michelson's value, determined in 1892–3, and that obtained by Fabry in 1912, with an interferometer of different principle and construction, forms one of the great triumphs of scientific measurement. Michelson's value of the wave length of the red cadmium line was $0{\cdot}64384722\mu$, while Fabry's was $0{\cdot}64384696\mu$. Allowing for the probable humidity of the air (not taken into account by Michelson) and for a slight adjustment proper to slightly different thermometric scales employed, the two determinations become almost identical.

Chapter III deals (a little too briefly) with "la mesure des petits angles," under which placid title is included Michelson's audacious and successful attempt to measure stellar diameters.

Chapter IV, "Applications à la Spectroscopie," consists of forty pages, not one of which could well be spared in setting forth this important section of interferometry.

The reviewer has noticed only one mistake (Fig. 55, indicating the passage of light through a Michelson echelon in the direction contrary to that described by Michelson and in general usage), and very little indeed to which exception could be taken. The book is likely to be for years the best résumé of this important subject.

There is a valuable bibliography, consisting of 141 references, though not all of them are correct.

F. T.

La Lumière Monochromatique sa Production et son Emploi en Optique Pratique. By CH. FABRY. Pp. 37; 16 Figs. (Paris: Published by the Revue d'Optique théorique et instrumentale, 1923.) Price, 3 fr.

This useful little book contains comprehensive information relative to the production of monochromatic light for the purposes of technical optics. Very little is contained in it that is not also contained in the book by Professor Fabry published in the same series (Editions de la Revue d'Optique théorique et instrumentale), notice of which appears above; but having in view that the smaller book should have great interest for many opticians who are not likely to be concerned with the more complex and purely scientific applications of interference, the collection of the information in a small separate handbook is probably worth while.

The book contains information on the production of monochromatic light by flames, vacuum tubes (including mercury vapour lamp), the metallic arc and spark, and also by selection from white light of a monochromatic region.

In the section devoted to applications of monochromatic light one finds interference by thin plates, by thick plates, and interferential methods of test for the aberrations of optical elements. Three pages only are devoted to the employment of monochromatic light in the study of optical properties of substances; and it is an unfortunate omission that there is no bibliography through which the earnest enquirer, unfamiliar with the literature on the subject, might find further information. But the book is none the less one that should be in the hands of every manufacturing optician who wishes to keep in touch with modern methods of test and modern applications of optics.

F. T.